Analysis of Abelian gauge theory with four-Fermi interaction at $O(1/N^2)$ in arbitrary dimensions

1993 J. Phys. A: Math. Gen. 26 1431

(http://iopscience.iop.org/0305-4470/26/6/024)

J. Phys. A: Math. Gen. **26** (1993) 1431–1440. Printed in the UK

Analysis of Abelian gauge theory with four-Fermi interaction at O($1/N^2$) in arbitrary dimensions

J A Gracey

Department of Applied Mathematics and Theoretical Physics, University of Liverpool, PO Box 147, Liverpool, L69 3BX, UK

Received 26 August 1992

Abstract. An arbitrary dimensional expression is given for the anomalous dimension of the fermion field in a model with a four-point interaction and a $U(1)$ gauge field, at O($1/N^2$) within a large flavour expansion in the Landau gauge.

1. Introduction

The use of the conventional large-N expansion to analyse (renormalizable) quantum field theories has proved to be successful in revealing properties which would otherwise be inaccessible in conventional perturbation theory. The method relies, essentially, on expanding the effective action in the saddle point approximation, which then determines the leading order structure in $1/N$. It is virtually impossible, however, to push that analysis beyond the leading order and probe corrections at O($1/N^2$). One method of overcoming this shortcoming was developed in [1, 2] for the O(N) bosonic σ model, and applied to other models in [3–5]. It is based on solving the skeleton Dyson equations at the d-dimensional critical point of the theory. A major simplification of this approach is the use of massless fields at this conformal point of the theory. Hence, one can systematically solve for the critical exponents, which depend only on N and the spacetime dimension, to several orders beyond the first which represents significant progress. Further, the absence of a mass allows one to make use of the powerful technique of uniqueness, introduced in [6], to evaluate the integrals which arise at higher orders.

Whilst the initial application of this self-consistency method was to a bosonic model, the techniques have also been developed to probe fermionic theories, such as the Gross Neveu model [3] and quantum electrodynamics (QED) [7], both to O($1/N^2$) by computing the critical exponents corresponding to the anomalous dimension of the basic fermionic field. Other critical exponents corresponding to the anomalous dimensions of the 3-vertices, as well as the β-function, have also been deduced [8]. More recently, we examined a fermionic model involving a four-Fermi interaction coupled to a $U(1)$ gauge field at leading order, O($1/N$), which is an amalgam of the Gross Neveu model and QED [8]. Currently, there is renewed interest in understanding four-Fermi interactions [9] in addition to earlier work [10], in an attempt to provide insight into the possible composite nature of the Higgs boson in the standard model [11]. In this paper, we therefore extend the O($1/N$) analysis of [8] by computing the O($1/N^2$) corrections to the critical exponent corresponding to the fermion anomalous

0305-4470/93/061431+10\$07.50

dimension in arbitrary dimensions in the Landau gauge, which will provide a more realistic scenario for determining the effects the presence of a gauge field has on the anomalous dimension, rather than studying four-Fermi interactions in isolation. Aside from this motivation, such a calculation will serve as a basis for a future discussion of the Nambu Jona-Lasinio model coupled to QED, (see, for example [12]). Further, we will gain insight into the three-dimensional structure of the model, which can be deduced simultaneously from the present calculation and which has been examined in [13].

The paper is structured as follows. In section 2, we introduce our basic formalism and review the leading order critical point analysis for the model we are interested in, which we formally extend to $O(1/N^2)$ in section 3. We discuss the techniques to evaluate the relevant two-loop massless Feynman integrals in section 4 and present our results in section 5.

2. Basic formalism

We begin by introducing the formalism required to solve the Schwinger-Dyson equation at criticality. First, the (massless) Lagrangian we use is [8]

$$L = \mathrm{i}\bar{\psi}^i \partial\!\!\!/ \psi^i - \frac{(F_{\mu\nu})^2}{4e^2} + A_\mu \bar{\psi}^i \gamma^\mu \psi^i + \rho \bar{\psi}^i \psi^i - \frac{\rho^2}{2g^2} \tag{2.1}$$

where $1 \leqslant \mathrm{i} \leqslant N$, with N playing the role of our expansion parameter, A_μ is the $U(1)$ gauge field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the coupling constants e and g appear in their respective kinetic terms, in anticipation of the method we will use [1, 2]. The four-Fermi interaction can be introduced explicitly by eliminating the auxiliary field, ρ, by its equation of motion, though we will use the Lagrangian (1) since the method of [1, 2] applies only to theories with three-point interactions. As in previous models we introduce the general structure of the fields at the critical point of the theory, consistent with Lorentz symmetry, in coordinate space, where the fields are massless. As our model involves a $U(1)$ gauge field, we choose to work in a particular (covariant) gauge, the Landau gauge. This choice is motivated by the nature of the large N expansion, which is a reordering of perturbation theory such that chains of bubbles are summed first. Thus one must be careful to choose a gauge, which is neither affected by this resummation nor altered by perturbative renormalization effects [14]. In the critical region, we therefore take the following asymptotic scaling forms for the propagators of the three fields of (2.1), as [1, 3, 15]

$$\begin{gathered} \psi(x) \sim \frac{A x\!\!\!/}{(x^2)^\alpha} \qquad \rho(x) \sim \frac{B}{(x^2)^\beta} \\ A_{\mu\nu}(x) \sim \frac{C}{(x^2)^\gamma} \left[\eta_{\mu\nu} + \frac{2\gamma}{(2\mu - 2\gamma - 1)} \frac{x_\mu x_\nu}{x^2} \right] \end{gathered} \tag{2.2}$$

as $x \to 0$, where A, B and C are the amplitudes of the respective fields, α, β and γ are their critical exponents and $d = 2\mu$ is the dimension of spacetime. Using the Fourier transform

$$\frac{1}{(x^2)^\alpha} = \frac{a(\alpha)}{\pi^\mu 2^{2\alpha}} \int_k \frac{\mathrm{e}^{\mathrm{i}kx}}{(k^2)^{\mu-\alpha}} \tag{2.3}$$

and its derivatives, where k is the conjugate momentum and $a(\alpha)=\Gamma(\mu-\alpha)/\Gamma(\mu)$, it is easy to check that (2.2) indeed possesses the usual Landau gauge structure, $P_{\mu\nu}(k)=\eta_{\mu\nu}-k_\mu k_\nu/k^2$. From a dimensional analysis of the action with Lagrangian (2.1) we introduce the anomalous pieces of each exponent by further defining

$$\alpha=\mu+\tfrac{1}{2}\eta \qquad \beta=1-\eta-\chi_\rho \qquad \gamma=1-\eta-\chi_A \tag{2.4}$$

where η is the (gauge-dependent) fermion anomalous dimension which we determine at $O(1/N^2)$ here. The remaining relations are deduced from the dimensions of the 3-vertices of the action with (2.1) as its Lagrangian, where χ_ρ and χ_A are the anomalous dimensions, respectively, of the interactions involving ρ and A_μ. In previous work [3] we computed each at $O(1/N)$ by carrying out a leading order renormalization precisely at the critical point using a method based on [16]. This preliminary analysis was necessary for providing independent checks on the present calculation. We record that with $\eta=\Sigma_{i=1}^{\infty}\eta_i/N^i$

$$\eta_1=\frac{(4\mu^2-10\mu+5)\Gamma(2\mu-1)}{4\Gamma^3(\mu)\Gamma(2-\mu)} \tag{2.5}$$

$$\chi_{\rho 1}=-\frac{(4\mu^2-6\mu+3)}{(4\mu^2-10\mu+5)}\eta_1 \tag{2.6}$$

$$\chi_{A1}=-\eta_1 \tag{2.7}$$

with the convention, tr $1=4$.

In addition to (2.2) we will require the asymptotic scaling forms of the two-point functions, ψ^{-1}, ρ^{-1} and $A^{-1}_{\mu\nu}$. These are derived from (2.2), by inverting those functions in momentum space before mapping back to coordinate space. Thus [1, 3, 8]

$$\psi^{-1}(x)\sim\frac{r(\alpha-1)\not{x}}{A(x^2)^{2\mu-\alpha-1}} \qquad \rho^{-1}(x)\sim\frac{p(\beta)}{B(x^2)^{2\mu-\beta}}$$
$$A^{-1}_{\mu\nu}(x)\sim\frac{m(\gamma)}{C(x^2)^{2\mu-\gamma}}\left[\eta_{\mu\nu}+\frac{2(2\mu-\gamma)}{(2\gamma-2\mu-1)}\frac{x_\mu x_\nu}{x^2}\right] \tag{2.8}$$

where

$$p(\alpha)=\frac{a(\alpha-\mu)}{\pi^{2\mu}(\alpha)} \qquad r(\alpha)=\frac{\alpha p(\alpha)}{(\mu-\alpha)} \qquad m(\gamma)=\frac{[4(\mu-\gamma)^2-1]p(\gamma)}{4(\mu-\gamma)^2}. \tag{2.9}$$

To obtain the expression for $A^{-1}_{\mu\nu}(x)$ we have first transformed $A_{\mu\nu}(x)$ to momentum space using (2.3) and inverted it on the transverse subspace, since this is the physically important part [16, 17], before mapping back to coordinate space.

As a preliminary to our $O(1/N^2)$ analysis we illustrate the method by deriving η_1 at leading order in coordinate space. First, the skeleton Dyson equations, with dressed propagators, for each field of (2.1) are illustrated in figures 1–3 . For the moment we will ignore the two-loop corrections and concentrate on the one-loop graphs. As each equation is valid in the critical region, we can represent them by replacing the lines of each graph by (2.2) and (2.8), since these will dominate. Thus,

$$0=r(\alpha-1)+z+\frac{2(2\mu-1)(\gamma-\mu+1)y}{(2\mu-2\gamma-1)} \tag{2.10}$$

$$0=p(\beta)+4Nz \tag{2.11}$$

$$0=m(\gamma)\left[\eta_{\mu\nu}+\frac{2(2\mu-\gamma)}{(2\gamma-2\mu-1)}\frac{x_\mu x_\nu}{x^2}\right]-4Ny\left[\eta_{\mu\nu}-\frac{2x_\mu x_\nu}{x^2}\right] \tag{2.12}$$

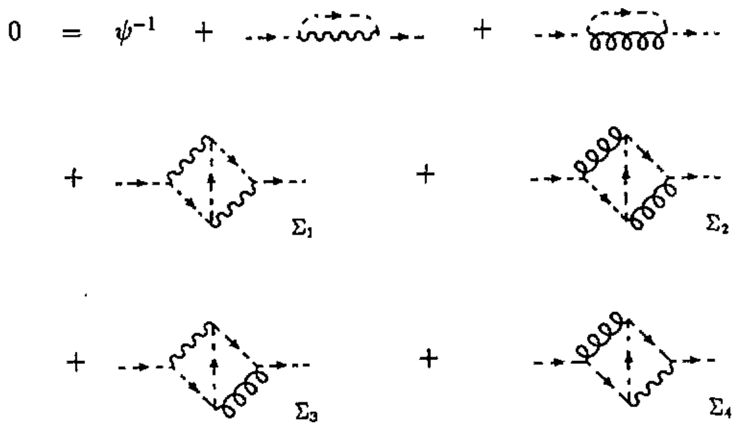

Figure 1. Skeleton Dyson equation with dressed propagators for ψ.

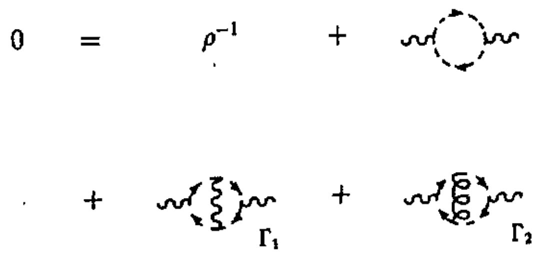

Figure 2. Skeleton Dyson equation with dressed propagators for ρ.

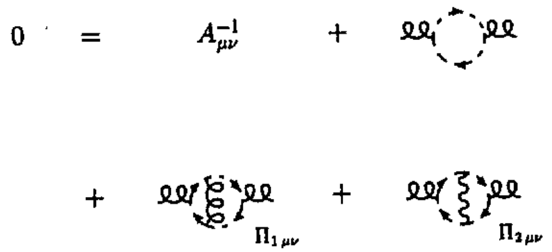

Figure 3. Skeleton Dyson equation with dressed propagators for the gauge field.

where $z = A^2B$ and $y = A^2C$. To isolate the physically relevant part of the photon equation we map (2.12) to momentum space and project out with the operator $P_{\mu\nu}(k)$ before mapping back again to give,

$$0 = \frac{(\gamma-\mu)m(\gamma)}{(2\gamma-2\mu-1)} - \frac{4(\alpha-1)Ny}{(2\alpha-1)}. \tag{2.13}$$

As the only unknowns are η, y and z to leading order, eliminating the latter two from (2.10), (2.11) and (2.13) yields (2.5), whence

$$y_1 = -\frac{(2\mu-3)\Gamma(2\mu)}{16\Gamma(\mu)\Gamma(2-\mu)\pi^{2\mu}} \qquad z_1 = -\frac{\Gamma(2\mu-1)}{4\Gamma(1-\mu)\Gamma(\mu-1)\pi^{2\mu}}. \tag{2.14}$$

The important part of this exercise aside from illustrating the simplicity of the technique over the conventional large N renormalization, is to provide the foundation for proceeding to higher orders. In part this entails expanding each term of (2.10)-(2.12) to a subsequent order and also including the higher order graphs of figures 1-3.

3. Corrections to basic formalism

The procedure to include the higher order two-loop graphs of figures 1-3 is not straightforward. Whilst it is still valid to represent the lines of the graphs by (2.8), it follows from their explicit evaluation, which we discuss later, that the graphs are infinite. These infinities arise from the presence of divergent vertex subgraphs when the vertex anomalous dimensions are zero. To handle this we introduce a regulator by shifting the exponents of both the gauge field and ρ by the infinitesimal quantity Δ, setting $\beta \to \beta - \Delta$ and $\gamma \to \gamma - \Delta$. Consequently, a renormalization procedure is required which will give a finite set of corrected consistency equations. We illustrate this in detail for the fermion whose Dyson equation is given in figure 1. First, we denote by Σ_i the value of the respective higher order graphs, by which we mean that function of the exponents obtained by computing the integral with unit amplitudes and without symmetry factors. (We note that Furry's theorem for (2.1) has excluded various three-loop graphs which are non-zero at $O(1/N^2)$ in other models.) Then we represent figure 1 near criticality as

$$0 = r(\alpha - 1) + zu^2(x^2)^{\chi_\rho + \Delta} + yv^2 f(\gamma - \Delta)(x^2)^{\chi_A + \Delta} + z^2(x^2)^{2\chi_\rho + 2\Delta}\Sigma_1 + y^2(x^2)^{2\chi_A + 2\Delta}\Sigma_2 + 2yz(x^2)^{\chi_\rho + \chi_A + 2\Delta}\Sigma_3 \tag{3.1}$$

where $f(\gamma) = 2(2\mu - 1)(\gamma - \mu + 1)/(2\mu - 2\gamma - 1)$ and we note that we have not cancelled the powers of x^2 since they contribute at $O(1/N^2)$. We have set $\Sigma_3 = \Sigma_4$ which can be observed from the explicit calculation or by making a change of variables in the integral itself. In (3.1) we have introduced the vertex counterterms u and v for $\rho\bar{\psi}\psi$ and $A_\mu \bar{\psi}\gamma^\mu\psi$, respectively, which will be required for removing the infinities. To display these divergences explicitly we define

$$\Sigma_i = \frac{K_i}{\Delta} + \Sigma_i' \tag{3.2}$$

where the prime, $'$, denotes the purely finite part of Σ_i with respect to Δ. To fix the counterterm we isolate all $1/\Delta$ pieces contributing to (3.1) by expanding each term in powers of Δ and setting this to zero, i.e.

$$0 = 2u_1 z_1 + \frac{4(2\mu - 1)(2 - \mu)v_1 y_1}{(2\mu - 3)} + \frac{z_1^2 K_1}{\Delta} + \frac{y_1^2 K_2}{\Delta} + \frac{2y_1 z_1 K_3}{\Delta} \tag{3.3}$$

which corresponds to a minimal choice. Absorbing a finite piece will not affect the value of any exponent we compute. Whilst this choice allows us to set $\Delta \to 0$ we are still unable to probe the critical region $x \to 0$ due to the presence of $\ln x^2$ terms which arise from the non-cancellation of powers of x^2 in (3.1). To remove these singular terms one exploits the freedom in choosing the as yet unspecified vertex anomalous dimensions. Thus defining

$$z_1 \chi_{\rho 1} + f(\gamma) y_1 \chi_{A1} = -[z_1^2 K_1 + y_1^2 K_2 + 2y_1 z_1 K_3] \tag{3.4}$$

we obtain the formal finite representation of the Dyson equation valid in the critical region as

$$0=r(\alpha-1)+z+f(\gamma)y-2f'(\gamma)y(v-1)\Delta+z^2\Sigma_1'+y^2\Sigma_2'+2yz\Sigma_3'. \qquad (3.5)$$

One check on this renormalization will come from substituting the explicit values for the poles into (3.4) and showing it agrees with (2.6) and (2.7).

The analysis for ρ proceeds along analogous lines and its finite consistency equation is

$$0=\frac{p(\beta)}{N}+4z-z^2\Gamma_1'-yz\Gamma_2' \qquad (3.6)$$

where $\Gamma_i=G_i/\Delta+\Gamma_i'$ and we have defined

$$\chi_{\rho 1}=\tfrac{1}{4}[z_1g_1+y_1G_2]. \qquad (3.7)$$

The minus signs in (3.6) arise from the factor associated with a fermion loop.

The treatment of the A_μ field, however, requires more care. First, the Dyson equation of figure 3 is represented by

$$0=\frac{m(\gamma-\Delta)}{N(x^2)^{2\mu-\gamma+\Delta}}\left[\eta_{\mu\nu}+\frac{2(2\mu-\gamma+\Delta)}{(2\gamma-2\mu-1-2\Delta)}\frac{x_\mu x_\nu}{x^2}\right]$$
$$-\frac{4yv^2}{(x^2)^{2\alpha}}\left[\eta_{\mu\nu}-\frac{2x_\mu x_\nu}{x^2}\right]-\frac{y^2\Pi_{1\mu\nu}}{(x^2)^{4\alpha+\gamma-2\mu-2-\Delta}}-\frac{yz\Pi_{2\mu\nu}}{(x^2)^{4\alpha+\beta-2\mu-2-\Delta}}. \qquad (3.8)$$

To obtain the correction to (2.13) from (3.8) we first map (3.8) to momentum space and project out the physically relevant transverse component before cancelling off a common power of x^2 [7], and then proceed with the renormalization. Defining $\Pi_{i\mu\nu}=\Pi_i\eta_{\mu\nu}+\Xi_i x_\mu x_\nu/x^2$ and the residues at the Δ-poles as P_i and X_i, respectively, we have

$$0=\frac{2(\mu-\gamma)m(\gamma)}{(2\mu-2\gamma+1)N}-\frac{8(\alpha-1)y}{(2\alpha-1)}-y^2\left(\Pi_1'+\frac{\Xi_1'}{2(2\alpha-1)}+\frac{X_1}{2(2\alpha-1)^2}\right)$$
$$-yz\left(\Pi_2'+\frac{\Xi_2'}{2(2\alpha-1)}+\frac{X_2}{2(2\alpha-1)^2}\right) \qquad (3.9)$$

where

$$\chi_{A1}=-\frac{(2\alpha-1)}{8(\alpha-1)}\left[y_1\left(P_1+\frac{X_1}{2(2\alpha-1)}\right)+z_1\left(P_2+\frac{X_2}{2(2\alpha-1)}\right)\right]. \qquad (3.10)$$

The appearance of the residues, X_i, in (3.9) arise from the Δ-expansion of a function of the exponents which enters after one has projected out the momentum space component. Of course, the same vertex counterterms u and v of (3.1) appear in the respective one-loop graphs of figures 2 and 3. The procedure to complete the calculation is the same as at leading order. One eliminates y and z from (3.5), (3.6) and (3.9) to leave one equation with η_2 as the only unknown and then substitutes the explicit values for the two-loop graphs.

4. Computation of two-loop graphs

Whilst several of the integrals in (3.5), (3.6) and (3.9) have already been computed in the constituent models of (2.1), we require the values for the mixed graphs. To compute these we apply the method of uniqueness of [6] and the method of subtractions of [1, 2]. For completeness, we will illustrate the procedure by computing $\Pi_{2\mu\nu}$ as an example. The basic uniqueness integration rule we require for a $\sigma\bar{\psi}\psi$ vertex is displayed in figure 4, where α_i are arbitrary exponents, satisfying the uniqueness constraint $\Sigma_{i=1}^{3}\alpha_i=2\mu+1$ and $\nu(\alpha_1,\alpha_2,\alpha_3)=\pi^{\mu}\Pi_{i=1}^{3}a(\alpha_i)$. To show the necessity of a regulator, if we set $\Delta=0$ in $\Pi_{2\mu\nu}$ and use the rule of figure 4 to complete the first integral, we obtain a final value involving the ill-defined quantity $a(\mu)$ which corresponds to the infinity mentioned earlier. However, the regularized graph cannot be integrated exactly with this rule since the vertex is no longer unique. Whilst this makes a total evaluation of $\Pi_{2\mu\nu}$ impossible for all Δ it is important to realise that we only need the pole and finite terms with respect to Δ for η_2. To extract these we subtract a quantity, $A(\Delta)$, from $\Pi_{2\mu\nu}(\Delta)$ which has the same infinity structure but which can be integrated explicitly for non-zero Δ. Then the difference $\Pi_{2\mu\nu}(\Delta)-A(\Delta)$ will be finite and therefore calculable at $\Delta=0$. The integral which satisfies these criteria is

$$
\begin{aligned}
A(\Delta)=&\int_y\int_z \rho\Delta(y-z)\,\mathrm{tr}[\gamma^{\mu}\not\psi(-y)\not\psi(y-x)\gamma^{\nu}\not\psi(x-y)\not\psi(z)]\\
&+\int_y\int_z \rho\Delta(y-z)\,\mathrm{tr}[\gamma^{\mu}\not\psi(-z)\not\psi(y-x)\gamma^{\nu}\not\psi(x-z)\not\psi(z)]
\end{aligned}
\tag{4.1}
$$

where y and z are the locations of the internal vertices of integration. Each term of (4.1) can be determined by first integrating with respect to y for non-zero Δ. The final integration is simplified by the general result

$$
\begin{aligned}
\int_y &\frac{\mathrm{tr}[\gamma^{\mu}(-\not y)(\not y-\not x)\gamma^{\nu}(\not x-\not y)\not y]}{(y^2)^{\alpha_1}((x-y)^2)^{\alpha_2}}\\
&=\frac{4\nu(\alpha_1-1,\alpha_2-1,2\mu-\alpha_1-\alpha_2+2)}{(\alpha_1-1)(\alpha_2-1)}\\
&\quad\times\left[[(\alpha_1-2)(\alpha_2-2)+\mu-1]\eta^{\mu\nu}-2(\mu-1)(\alpha_1+\alpha_2-\mu-2)\frac{x_{\mu}x_{\nu}}{x^2}\right]
\end{aligned}
\tag{4.2}
$$

which is valid for all α_i. Thus adding the finite piece to $A(\Delta)$, we find

$$
\Pi_{2\mu\nu}=\frac{8\pi^{2\mu}}{\mu\Gamma^2(\mu)}\left[\frac{1}{\Delta}-\frac{1}{(\mu-1)}\right]\left[\eta_{\mu\nu}-\frac{2x_{\mu}x_{\nu}}{x^2}\right]
\tag{4.3}
$$

where we have set the exponents to their leading order large N values, $\alpha=\mu$, $\beta=\gamma=1$. It is worth noting that when one transforms (4.3) to momentum space in the context of (3.8), the usual projection operator $P_{\mu\nu}(k)$ emerges, which confirms that our result is gauge invariant. The procedure to compute the remaining mixed graphs is similar

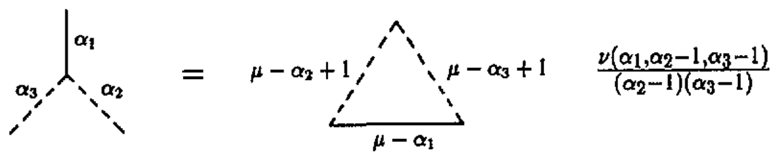

Figure 4. Integration rule for $\Pi_{2\mu\nu}$.

to that for $\Pi_{2\mu\nu}$ and we merely quote our results for each to limit the repetitive nature of the discussion. We found

$$\begin{aligned}\Sigma_3 &= -\frac{4\pi^{2\mu}(2\mu-1)(\mu-1)}{\mu(2\mu-3)\Gamma^2(\mu)}\left[\frac{1}{\Delta}+\frac{(\mu^2-\mu+2)}{2\mu(\mu-1)^2}-\frac{2}{(2\mu-3)}\right] \\ \Gamma_2 &= \frac{16\pi^{2\mu}}{(2\mu-3)\Gamma^2(\mu)}\left[\frac{(2\mu-1)}{\Delta}-\frac{2(2\mu-1)}{(2\mu-3)}+\frac{3}{(\mu-1)}-3(\mu-1)\hat{\Theta}(\mu)\right]\end{aligned} \tag{4.4}$$

where $\hat{\Theta}(\mu)=\psi'(\mu-1)-\psi'(1)$ and $\psi(\mu)$ is the logarithmic derivative of the Γ-function. For the integrals involving an internal gauge field we had to first apply an integration by parts rule which was introduced in [7]. This allows one to rewrite the integral in terms of simpler integrals where it is easier to take the trace over eight γ-matrices, to yield a set of relatively simple bosonic two-loop graphs to which one can apply the subtraction procedure of [2].

Finally, for completeness we present the results obtained for the remaining integrals, noting that the additional factor of 2 which appears in Γ_1 compared with the analogous integral in [3] is due to our convention that $\mathrm{tr}1=4$. Thus,

$$\begin{aligned}\Sigma_1 &= -\frac{2\pi^{2\mu}}{(\mu-1)\Gamma^2(\mu)\Delta}\left[1-\frac{\Delta}{2(\mu-1)}\right] \\ \Sigma_2 &= \frac{4\pi^{2\mu}(2\mu-1)}{(2\mu-3)^2\mu\Gamma^2(\mu)}\left[\frac{2(2\mu-1)(\mu-2)^2}{\Delta}+(2\mu-5)\mu\right. \\ &\qquad\left.+\frac{4(\mu-1)^2(\mu-2)}{\mu}-\frac{8(2\mu-1)(\mu-2)^2}{(2\mu-3)}\right] \\ \Gamma_1 &= \frac{8\pi^{2\mu}}{(\mu-1)\Gamma^2(\mu)\Delta}\left[1-\frac{\Delta}{(\mu-1)}\right] \\ \Pi_{1\mu\nu} &= \frac{16\pi^{2\mu}}{(2\mu-3)\Gamma^2(\mu)}\left[\eta_{\mu\nu}-\frac{2x_\mu x_\nu}{x^2}\right] \\ &\qquad\times\left[\frac{(2\mu-1)(2-\mu)}{\mu\Delta}+3(\mu-1)\hat{\Theta}(\mu)-\frac{3}{(\mu-1)}+\frac{2(2\mu-1)(\mu-2)}{(2\mu-3)\mu}\right].\end{aligned} \tag{4.5}$$

These have been computed earlier in the separate models, [3, 7], using the technique described at the start of this section.

5. Discussion

With these explicit values we can now complete our $O(1/N^2)$ analysis. First, isolating the residues at the Δ-poles we have verified that the vertex anomalous exponents of (2.6) and (2.7) re-emerge, which is a partial check on our renormalization and more importantly on the computation of the two-loop graphs. Further, each exponent is consistent with (3.4). It is worth noting that the relatively simple result, $\chi_{1A}=-\eta_1$ follows directly from the Ward identity for the photon field which is similar to what occurs in the pure QED version [7]. More importantly, we can eliminate y_2 and z_2 from our finite consistency equations and after some straightforward algebra we found the following arbitrary dimensional expression for η_2,

$$\eta_2=\frac{(2\mu-1)S_d^2}{4\mu(\mu-1)^2}\left[4(\mu-1)^2\hat{\Psi}+16\mu^2(\mu-4)+11(9\mu-5)+\frac{4}{(\mu-2)}\right.$$
$$\left.+\frac{8}{\mu}+3\mu(\mu-1)(4\mu^2-10\mu+5)\left(\hat{\Theta}-\frac{1}{(\mu-1)^2}-\frac{(2\mu-1)}{3\mu(\mu-1)}\right)\right] \quad (5.1)$$

where we have defined $\hat{\Psi}(\mu)=\psi(2\mu-2)+\psi(3-\mu)-\psi(1)-\psi(\mu)$ and $S_d=a(2-\mu)a^2(\mu-1)/\Gamma(\mu)$.

We conclude with several observations. First, the result (5.1) can be evaluated in three dimensions to gain new information about that model, and we find

$$\eta=-\frac{2}{\pi^2N}+\frac{4}{9\pi^4N^2}[142-9\pi^2]+O\left(\frac{1}{N^3}\right). \quad (5.2)$$

With (5.2) we can now examine the question of whether the $O(1/N^2)$ correction gives a significant numerical contribution to the value of the exponent. The approximation we have used to solve (2.1) is to assume N is large. However, in the conventional approach which determines information solely at leading order it is never clear for which range of values of N one can obtain reliable accurate information. For instance, the next to leading order corrections may be of the same magnitude as leading order for a large range of N and so the approximation would not be a sensible one. By providing an exact expression at $O(1/N^2)$ one can gain a much better picture of the convergence of the series in $1/N$ which is one justification for considering the next to leading order. For instance, from (5.2) it turns out that the lowest value of N for which there is a sensible correction is $N=3$ which is smaller than one would naively expect for a large N approximation. In light of these remarks we can now compare (5.2) with η in the Gross Neveu model and QED for $N=3$ to ascertain which constituent model governs the quantum properties of the fermion field. So being careful to have the same trace convention for each case, we find the respective values for η are -0.04, 0.06 and -0.09. Thus, it would appear that the dominant contribution in three dimensions derives from the QED sector. Further, it is important to note that information concerning the three-dimensional model, (2.1), is presently of interest. Recently, several authors investigated the implications that a four-Fermi interaction coupled to QED has in relation to understanding high T_c superconductivity, [18]. Clearly it is necessary to deduce accurate information for such models and so to achieve this within the present formalism and to calculate other critical exponents, such as those relating to the β-function, to the *same* order as (5.1) the fundamental quantities y_2 and z_2 need to be known. These can be deduced from the relevant consistency equations now that η_2 is available. Indeed it is worth remarking that one of the features of our formalism is that the anomalous dimensions of the fundamental fields of a theory have to be computed first before attempting to calculate others.

Next, in four dimensions if one expands η_2 in powers of $\varepsilon=\mu-2$, then it appears that the leading contribution to the anomalous dimension of the amalgam model is from the four-Fermi interaction rather than the QED sector, since $\eta_2\sim\varepsilon$ for (1) and the Gross Neveu model, whilst $\eta_2\sim\varepsilon^2$ for QED, [7], similar to leading order. Finally, we remark that it would now be interesting to see if the equivalence of a Yukawa theory in $4-\varepsilon$ dimensions to a four-Fermi interaction in $2+\varepsilon$, which was established by Wilson in [10] and others in [9], is preserved by the addition of the $U(1)$ gauge field. In providing η for (1) at $O(1/N^2)$ here, we have introduced the first step in such an analysis.

References

[1] Vasil'ev A N, Pis'mak Yu M and Honkonen J R 1981 *Theor. Math. Phys.* **46** 157
[2] Vasil'ev A N, Pis'mak Yu M and Honkonen 1981 *Theor. Math. Phys.* **47** 291
[3] Gracey J A 1991 *Int. J. Mod. Phys.* A **6** 395
[4] Gracey J A 1991 *Nucl. Phys.* B **348** 737
[5] Gracey J A 1991 *Nucl. Phys.* B **352** 183
[6] d'Eramo M, Peliti L and Parisi G 1971 *Lett. Nuovo Cimento* **2** 878
[7] Gracey J A 1992 *Mod. Phys. Lett.* A **7** 1945
[8] Gracey J A 1992 *J. Phys. A: Math. Gen.* **25** L109
[9] Hands S J, Kocić A and Kogut J B 1991 *Phys. Lett.* **273B** 111;
Zinn-Justin J 1991 *Nucl. Phys.* B **367** 105
[10] Wilson K G 1973 *Phys. Rev.* D **7** 2911
[11] Bardeen W A, Hill C T and Lindner M 1990 *Phys. Rev.* D **41** 1467
Hasenfratz A, Hasenfratz P, Jansen K, Kuti J and Shen Y 1991 *Nucl. Phys.* B **365** 79
[12] Kenway R D 1989 *Rep. Prog. Phys.* **52** 1475
[13] Semenoff G W and Wijewardhana L C R 1992 *Phys. Rev.* D **45** 1341
[14] Palanques-Mestre A and Pascual P 1984 *Commun. Math. Phys.* **95** 277
[15] Vasil'ev A N, Pis'mak Yu M and Honkonen J R 1981 *Theor. Math. Phys.* **48** 750
[16] Vasil'ev A N and Nalimov M Yu 1982 *Theor. Math. Phys.* **55** 163; 1983 *Theor. Math. Phys.* **56** 15
[17] Slavnov A A and Faddeev L D 1978 *Introduction to the Quantum Theory of Gauge Fields* (Moscow: Nauka)
[18] Kovner A and Eliezer D 1992 *Int. J. Mod. Phys.* A **7** 2775